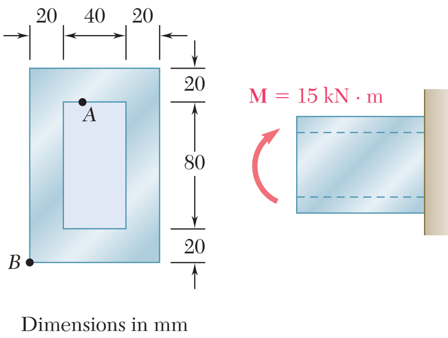


PROBLEM 4.1

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

For rectangle: $I = \dfrac{1}{12}bh^3$

Outside rectangle: $I_1 = \dfrac{1}{12}(80)(120)^3$

$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$

Cutout: $I_2 = \dfrac{1}{12}(40)(80)^3$

$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(*a*) $y_A = 40 \text{ mm} = 0.040 \text{ m}$

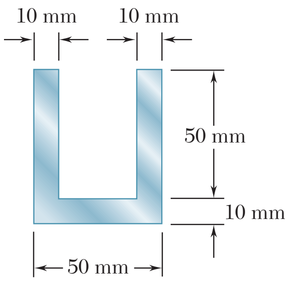
$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

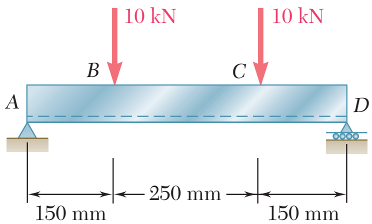
$\sigma_A = -61.6 \text{ MPa}$ ◀

(*b*) $y_B = -60 \text{ mm} = -0.060 \text{ m}$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$\sigma_B = 91.7 \text{ MPa}$ ◀

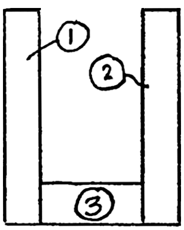




PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

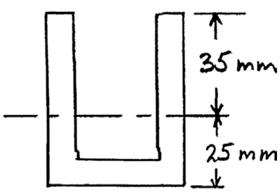
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \qquad I_2 = I_1 = 195\ \text{mm}^4$$

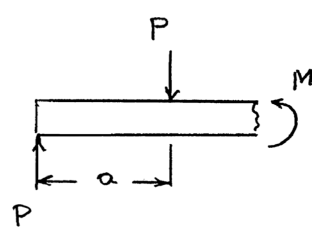
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$$y_{top} = 35\ \text{mm} = 0.035\ \text{m} \qquad y_{bot} = -25\ \text{mm} = -0.025\ \text{m}$$

$$a = 150\ \text{mm} = 0.150\ \text{m} \qquad P = 10\times10^3\,\text{N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$$

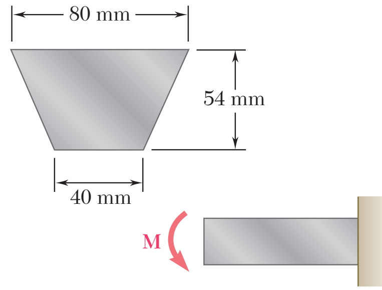


$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{top} = -102.4$ MPa (compression) ◀

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

$\sigma_{bot} = 73.2$ MPa (tension) ◀

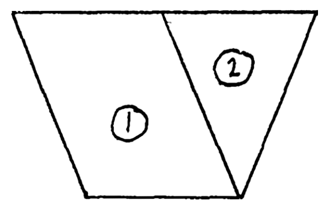


PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3	d, mm
①	2160	27	58,320	3
②	1080	36	38,880	3
Σ	3240		97,200	



$$\bar{Y} = \frac{97,200}{3240} = 30 \text{ mm}$$ The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \qquad y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32\times10^3 \text{mm}^4$$

$$I_2 = \frac{1}{36}b_2h_2^2 + A_2d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84\times10^3 \text{mm}^4$$

$$I = I_1 + I_2 = 758.16\times10^3 \text{mm}^4 = 758.16\times10^{-9} \text{m}^4$$

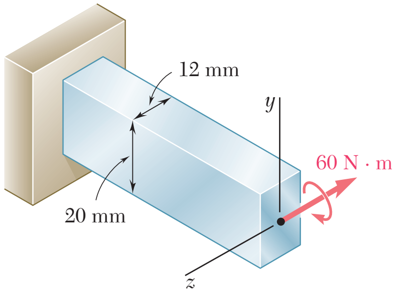
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(120\times10^6)(758.16\times10^{-9})}{0.024} = 3.7908\times10^3 \text{N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(150\times10^6)(758.16\times10^{-9})}{0.030} = 3.7908\times10^3 \text{N}\cdot\text{m}$$

Choose the smaller as M_{all}. $$M_{all} = 3.7908\times10^3 \text{N}\cdot\text{m}$$

$M_{all} = 3.79 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 4.24

A 60-N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

(*a*) <u>Bending about *z*-axis.</u>

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8\times10^3\,\text{mm}^4 = 8\times10^{-9}\,\text{m}^4$$

$$c = \frac{20}{2} = 10\text{ mm} = 0.010\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8\times10^{-9}} = 75.0\times10^6\,\text{Pa}$$

$\sigma = 75.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(8\times10^{-9})} = 37.5\times10^{-3}\,\text{m}^{-1}$$

$\rho = 26.7$ m ◀

(*b*) <u>Bending about *y*-axis.</u>

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88\times10^3\,\text{mm}^4 = 2.88\times10^{-9}\,\text{m}^4$$

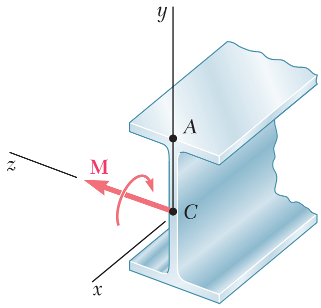
$$c = \frac{12}{2} = 6\text{ mm} = 0.006\text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88\times10^{-9}} = 125.0\times10^6\,\text{Pa}$$

$\sigma = 125.0$ MPa ◀

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(2.88\times10^{-9})} = 104.17\times10^{-3}\,\text{m}^{-1}$$

$\rho = 9.60$ m ◀



PROBLEM 4.31

A $W200 \times 31.3$ rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

SOLUTION

For $W200 \times 31.3$ rolled steel section,

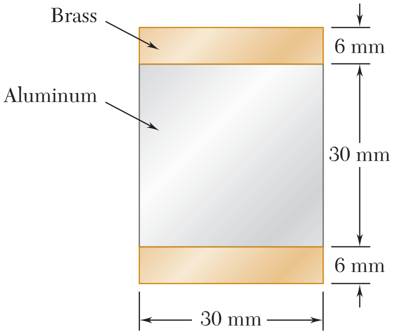
$$I = 31.3 \times 10^{6}\ \text{mm}^4$$
$$= 31.3 \times 10^{-6}\ \text{m}^4$$

(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.3 \times 10^{-6})} = 7.1885 \times 10^{-3}\ \text{m}^{-1}$$

$\rho = 139.1\ \text{m}$ ◀

(*b*) $$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.1885 \times 10^{-3}) = 2.0847 \times 10^{-3}\ \text{m}^{-1}$$

$\rho' = 480\ \text{m}$ ◀

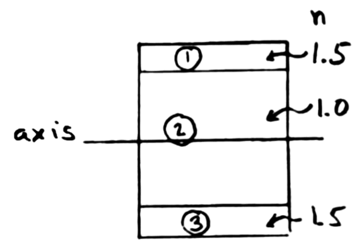


PROBLEM 4.33

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.5}{12}(30)(6)^3 + (1.5)(30)(6)(18)^3 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4$$

$$= 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{ny}$$

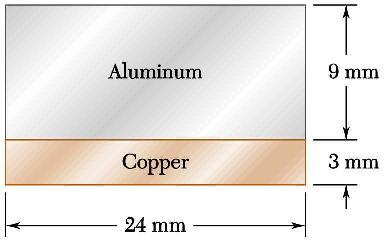
Aluminum: $n = 1.0, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad y = 21 \text{ mm} = 0.021 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value $M = 1.240 \times 10^3 \text{ N} \cdot \text{m}$ **1.240 kN · m ◀**



PROBLEM 4.40

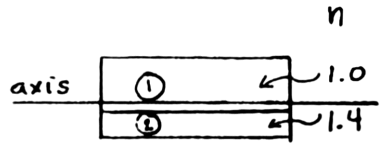
A copper strip ($E_c = 105$ GPa) and an aluminum strip ($E_a = 75$ GPa) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 35$ N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$

$$n = E_c/E_a = 105/75 = 1.4 \text{ in copper}$$



Transformed section:

	A, mm^2	nA, mm^2	$A\bar{y}_0$, mm	$nA\bar{y}_0$, mm^3
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_0 = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4839 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $n = 1.0, \quad y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^{-6} \text{ Pa}$$

$$= -56.0 \text{ MPa}$$

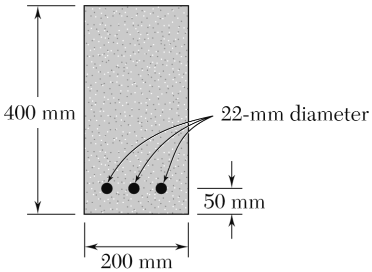
$\sigma = -56.0$ MPa ◀

(*b*) Copper: $n = 1.4, \quad y = -5.5909 \text{ mm} = -0.0055909 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^{6} \text{ Pa}$$

$$= 68.4 \text{ MPa}$$

$\sigma = 68.4$ MPa ◀



PROBLEM 4.52

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

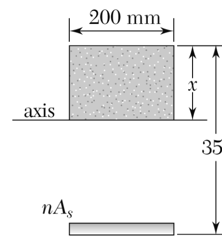
$$A_s = 3\frac{\pi}{4}d^2 = 3\frac{\pi}{4}(22)^2 = 1140 \text{ mm}^2$$

$$nA_s = 11400 \text{ mm}^2$$

Locate neutral axis

$$200\,x\frac{x}{2} - (11400)(350 - x) = 0$$

$$100x^2 + 11400x - 3,990,00 = 0$$



Solve for x:

$$x = \frac{-11400 + \sqrt{11400^2 + (4)(100)(3,990,000)}}{(2)(100)} = 150.72 \text{ mm}$$

$$350 - x = 199.28 \text{ mm}$$

$$I = \frac{1}{3}200x^3 + nA_s(350 - x)^2 = \frac{1}{3}(200)(150.72)^3 + (11400)(199.28)^2$$

$$= 681 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete:

$$n = 1.0, \quad |y| = 150.72 \text{ in}, \quad |\sigma| = 9 \text{ MPa}$$

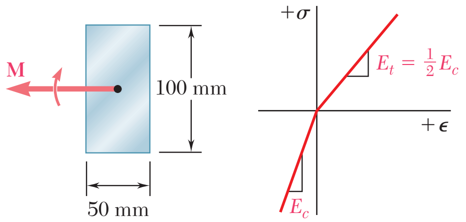
$$M = \frac{(9 \times 10^6)(681 \times 10^{-6})}{(1.0)(0.15072)} = 40664 \text{ N}\cdot\text{m} = 407 \text{ kN}\cdot\text{m}$$

Steel:

$$n = 10, \quad |y| = 199.28, \quad \sigma = 140 \text{ MPa}$$

$$M = \frac{(140\times10^6)(681\times10^{-6})}{(10)(0.19928)} = 478.42 \text{ N}\cdot\text{m} = 47.8 \text{ kN}\cdot\text{m}$$

Choose the smaller value. $M = 40.7 \text{ kN}\cdot\text{m}$ ◀



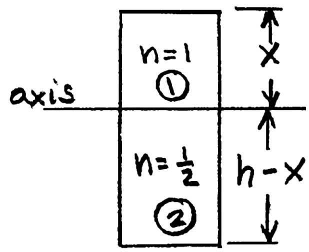
PROBLEM 4.59

The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$$n = \frac{1}{2} \text{ on the tension side of neutral axis}$$

$$n = 1 \text{ on the compression side}$$



Locate neutral axis.

$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421h = 41.421\ \text{mm}$$

$$h - x = 58.579\ \text{mm}$$

$$I_1 = n_1 \frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\ \text{mm}^4$$

$$I_2 = n_2 \frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\ \text{mm}^4$$

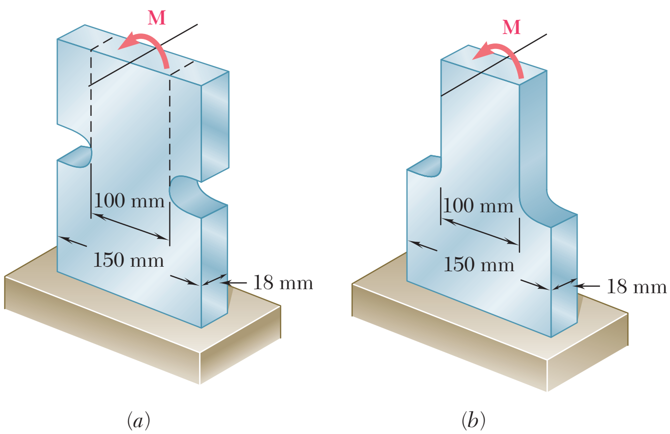
$$I = I_1 + I_2 = 2.8595\times10^6\ \text{mm}^4 = 2.8595\times10^{-6}\ \text{m}^4$$

(*a*) Tensile stress: $n = \frac{1}{2}, \quad y = -58.579\ \text{mm} = -0.058579\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\ \text{Pa} \qquad \sigma_t = 6.15\ \text{MPa} \blacktriangleleft$$

(*b*) Compressive stress: $n = 1, \quad y = 41.421\ \text{mm} = 0.041421\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\ \text{Pa} \qquad \sigma_c = -8.69\ \text{MPa} \blacktriangleleft$$



PROBLEM 4.65

A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material to the left and right of the dashed lines as shown in Fig. *b*.

SOLUTION

For both configurations,

$$D = 150 \text{ mm}$$
$$d = 100 \text{ mm}$$
$$r = 10 \text{ mm}$$
$$\frac{D}{d} = \frac{150}{100} = 1.50$$
$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (*a*),

Fig. 4.28 gives $K_a = 2.21.$

For configuration (*b*), Fig. 4.27 gives $K_b = 1.79.$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

(*a*) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147.0 \times 10^6 \text{ Pa} = 147.0 \text{ MPa}$$

$\sigma = 147.0$ MPa ◀

(*b*) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119.0 \times 10^6 \text{ Pa} = 119.0 \text{ MPa}$$

$\sigma = 119.0$ MPa ◀